Energy degradation and power generation

This chapter deals with energy sources and how the energy content of each source can be extracted. We make the important distinction between renewable (will not run out) and non-renewable (will run out) sources of energy. The chapter introduces the basic physics associated with nuclear fission, solar, wind, wave and wind power.

Objectives

By the end of this chapter you should be able to:

- explain the meaning of the term *energy degradation*;
- understand that, in the cyclic operation of an engine, *not all the available thermal energy can be transformed* to mechanical work;
- outline *how electricity is produced*;
- understand the difference between *renewable* and *non-renewable* forms of energy;
- state the meaning of the term *energy density*;
- understand how energy is produced by *nuclear fuels*;
- describe the function of the *main elements of a nuclear reactor*;
- appreciate the problems with *nuclear fusion*;
- understand the basics of *solar, wind, hydroelectric* and *wave power*;
- state the meaning of the term *solar constant*;
- discuss the relative *advantages* and *disadvantages* of various energy sources.

Degradation of energy

Thermal energy flows, as we have seen, from hot to cold bodies. The difference in temperature between two bodies offers the opportunity to run a 'heat engine' between those two temperatures, extracting useful mechanical work in the process. That is to say, some of the thermal energy that is transferred from the hot to the cold body can be transformed into mechanical work. This is shown in Figure 1.1. The diagram to the right in this figure is a **Sankey diagram** representing the energy flows. The *width* of each arrow in the diagram is proportional to the energy carried by that arrow. The input energy is 800 J and the useful mechanical work done is 200 J, giving an efficiency of

$$e = \frac{200}{800} = 0.25$$

With time, the two bodies will approach the same temperature, and the opportunity for using those two bodies to do work will be lost.

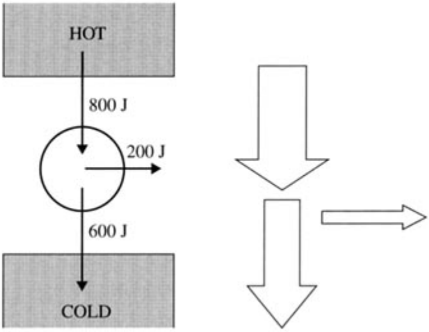


Figure 1.1 Schematic energy flow diagram for a heat engine. The diagram to the right is a Sankey diagram for the engine. The width of the arrow is proportional to the energy it carries.

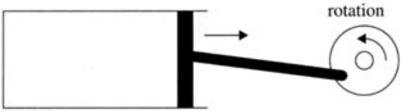
> ▶ Thus the flow of thermal energy from the hot to the colder body tends to equalize the two temperatures and deprives us of the opportunity to do work.

Suppose, then, that we do have two places ('reservoirs') at different temperatures. The thermal energy that can now flow from the hot to the colder reservoir can be used to perform work. The hot reservoir is the hot interior of the cylinders where gasoline or some other fuel is burned. The cold reservoir is the exhaust system of the engine. It must be understood that any *practical* heat engine must work in a *cycle*. That is to say, the engine begins in some state, absorbs thermal energy and does work. The engine must now be returned to its initial state so that the process can be *repeated*. For example, consider a gas that is kept in a container with a piston (Figure 1.2). The gas absorbs thermal energy and so expands. The motion of the piston can be used to perform mechanical work.

If all that happens is to expand the gas, then *all* the thermal energy absorbed can be transformed into mechanical work. To make the engine practical, the gas must be returned to its initial state (Figure 1.3), so that it can again absorb energy and perform more work. The piston must then be pushed back in to return the gas to its initial state.

The price to pay for operating in a cycle is that *not all* of the thermal energy transferred can be transformed into mechanical work. Some of this energy must be returned to the cold reservoir. While not lost, this energy is less useful. To use it to perform mechanical work, another *colder* reservoir must be found. This energy has become **degraded**. The necessity of losing some energy to the colder reservoir in a cyclic process is a consequence of a fundamental law of physics, the second law of thermodynamics.

(a) thermal energy absorbed



(b) gas expands, doing mechanical work

Figure 1.2 An expanding gas performs mechanical work.

(a) thermal energy absorbed

(b) gas expands, doing mechanical work

(c) gas returned to its initial stage, so that the cycle can be repeated – some thermal energy is ejected by the engine

Figure 1.3 The engine must work in a cycle and so must be returned to its initial state.

> ▶ Energy, while always being conserved, becomes less useful, i.e. it cannot be used to perform mechanical work – this is called *energy degradation*.

Electricity production

We are heavily dependent on electricity, and many of the energy sources that are available to us today are used to produce electricity. The production of electricity (almost universally) takes place in *electric generators*, such as the one described in Chapter 5.8. The main idea is to rotate a coil in a magnetic field so that magnetic field lines are cut by the moving coil. According to Faraday's law an emf (voltage) will be created in the coil, which can then be delivered to consumers. A generator thus converts mechanical energy (the rotational energy of the coil) into electrical energy. The various other sources of energy available are used to provide the mechanical energy of the rotating coil (Figure 1.4).

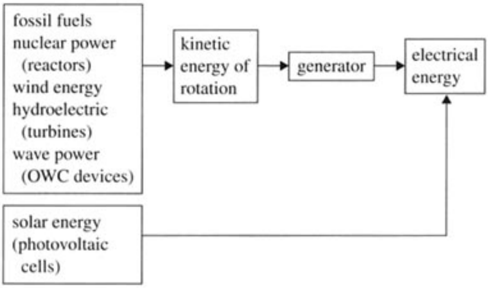


Figure 1.4 Methods of electricity production.

Energy sources

The development of civilization has gone hand in hand with an increase in the use of energy. Whereas ancient peoples had food and sunlight as their only sources of energy, consuming no more than about 8 MJ of energy per day, modern humans depend on energy for food, shelter, transportation, communication, manufacture of goods, services and entertainment. The average world use of energy per person amounts today to about 300 MJ per day in the more developed countries, giving a power consumption per person of about $300 \times 10^6/(24 \times 60 \times 60) = 3.5$ kW. In the USA, the power consumption per person is at a high of 10 kW, whereas in parts of Africa it is below 0.1 kW. The world average is about 0.8 kW per person. Taking 3 kW as an estimate for living in comfortable conditions, and a world population of 6.7 billion (6.7×10^9), results in a total world energy demand for one year of about 6.3×10^{20} J. This is very close to the total annual world production of energy, which is estimated to be about 1.5×10^{21} J. Taking into account that the world's population is increasing, it is obvious that we are faced with an extremely serious problem. We need *sources of energy*. We may classify energy sources into two large classes, **non-renewable** and **renewable**.

- *Non-renewable sources* of energy are finite sources, which are being depleted, and will run out. They include fossil fuels (e.g. oil, natural gas and coal) and nuclear fuels (e.g. uranium). The energy stored in these sources is, in general, a form of potential energy, which can be released by human action.
- *Renewable sources* include solar energy (and the other forms indirectly dependent on solar energy, such as wind energy and wave energy) and tidal energy.

The main sources of energy, and the percentage of the total energy produced of each, is given in Table 1.1. The figures are world averages and are approximate. Data for individual countries vary. World averages also vary, as different countries rely on different energy sources and change their dependence on any one particular fuel.

As mentioned earlier, most renewable sources are directly or indirectly linked to the sun.

Fuel	Percentage of total energy production (%)	Carbon dioxide emission (g MJ^{-1})
Oil	40	70
Natural gas	23	50
Coal	23	90
Nuclear	7	–
Hydroelectric	7	–
Others	<1	–

Table 1.1 Energy sources and the percentage of the total energy production for each. The third column gives the mass of carbon dioxide emitted per unit of energy produced from a particular fuel. Fossil fuels account for about 86% of the total energy production.

Energy density

A useful characteristic of fuels is their **energy density**.

> ▶ The *energy density* of a fuel is the energy that can be obtained from a unit mass of the fuel. Energy density is measured in J kg^{-1}.

If the energy is obtained by burning the fuel (as in fossil fuels), the energy density is simply the heat of combustion (see Table 1.2).

In a nuclear fission reaction, mass is converted *directly* into energy through Einstein's formula $E = mc^2$. As discussed in Chapter 6.3, one kilogram of uranium-235 releases a quantity of energy equal to 7×10^{13} J $= 7 \times 10^4$ GJ. Natural uranium (mainly uranium-238) contains about 0.7% of uranium-235, and so the energy density of natural uranium as a nuclear fuel is $\frac{0.7}{100} \times 7 \times 10^4 = 490$ GJ kg^{-1}, substantially higher than that of fossil fuels. Enriched uranium containing 3% uranium-235 has an energy density of $\frac{3}{100} \times 7 \times 10^4 = 2100$ GJ kg^{-1}.

Substance	Heat of combustion
Coal	30 MJ kg^{-1}
Wood	16 MJ kg^{-1}
Diesel oil	45 MJ kg^{-1}
Gasoline	47 MJ kg^{-1}
Kerosene	46 MJ kg^{-1}
Natural gas	39 MJ m^{-3} (at stp)

Table 1.2 Energy density of fossil fuels.

To calculate the energy density of water used in a hydroelectric power plant, imagine that 1 kg of water falls from a height of 100 m and that *all* the kinetic energy so gained is converted into the rotational motion that is used to produce electricity. The gain in kinetic energy is

$$\begin{aligned} E &= \frac{1}{2} mv^2 \\ &= mgh \\ &\approx 1 \times 10 \times 100 \\ &= 10^3 \text{ J} \end{aligned}$$

This implies that the energy density of water used as a 'fuel' in a hydroelectric power plant is 10^3 J kg^{-1}, substantially below the energy density of fossil and nuclear fuels.

Energy density is a major consideration in the choice of a fuel. Obviously, *all other factors being equal*, the higher the energy density, the more desirable the fuel.

Fossil fuels

Fossil fuels (oil, coal and natural gas) have been created over millions of years. They are produced by the decomposition of buried animal and plant matter under the combined action of the high pressure of the material on top and bacteria.

Burning coal and oil have been the traditional ways of producing electricity. The thermal energy released in combustion is used to power steam engines, which, in turn, power generators. Gasoline in internal combustion engines has been powering automobiles for over a century.

Although generally efficient (30–40%), these engines are primarily responsible for atmospheric pollution and contribute greenhouse gases to the atmosphere. In electricity-producing power plants using coal, the efficiency is typically around 30%, depending on the technology level of the plant and the precise cycles of operation. Natural gas produces somewhat higher efficiencies, typically 42%.

Example question

Q1

A power plant produces electricity by burning coal, using the thermal energy produced as input to a steam engine, which makes a turbine turn, producing electricity. The plant has a power output of 400 MW and operates at an overall efficiency of 35%.

(a) Calculate the rate at which thermal energy is provided by the burning coal.

(b) Hence calculate the rate at which coal is being burned (use a coal energy density of 30 MJ kg^{-1}).

(c) The thermal energy discarded by the power plant is removed by water (Figure 1.5). The temperature of the water must not increase by more than 5 °C. Calculate the rate at which the water must flow.

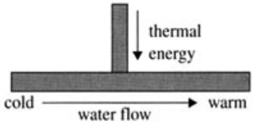


Figure 1.5.

Answer

(a) The efficiency is the ratio of power output to power input, and so thermal energy must be provided at a rate of

$$\frac{400}{0.35} = 1.14 \times 10^3 \approx 1.1 \times 10^3 \text{ MW}$$

(b) The amount of mass of coal that must be burned per second is found from

$$\frac{\Delta m}{\Delta t} \times 30 \times 10^6 = 1.14 \times 10^9 \Rightarrow \frac{\Delta m}{\Delta t} = 38 \text{ kg s}^{-1}$$

or

$$38 \times 60 \times 60 \times 24 \times 365 = 1.2 \times 10^9 \text{ kg yr}^{-1}$$

(c) The thermal energy discarded enters the water at a rate of

$$\frac{\Delta Q}{\Delta t} = 1.14 \times 10^3 - 0.400 \times 10^3 = 740 \text{ MW}$$

This thermal energy warms up the water according to $\Delta Q = (\Delta m)c\Delta T$, where Δm is the mass of water into which the thermal energy goes, c is the specific heat capacity of water (4200 J kg^{-1} K^{-1}) and ΔT is the temperature increase of the water (5 °C). The rate at which thermal energy enters the water is

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta m}{\Delta t} c\Delta T = 740 \text{ MW}$$

Thus, we find that

$$\frac{\Delta m}{\Delta t} = \frac{740 \times 10^6}{4200 \times 5}$$

$$= 35 \times 10^3 \text{ kg s}^{-1}$$

Fossil fuel mining

Coal is obtained by mining. The mining process produces a large number of toxic substances, and the coal itself, which is stored in large quantities near the mines, is high in sulphur content and traces of heavy metals. Rain can wash away the sulphur and heavy metal traces, creating serious environmental problems if this acidic water enters underground water reserves. The sites of coal-mining are also considered to be environmental disaster areas, which is why many countries have strict laws requiring mining companies to have plans for reclaiming the area after the mining is over. Drilling for oil also has adverse environmental effects, with many accidents leading to leakage of oil both at sea and on land.

▶ **Advantages of fossil fuels**

- Relatively cheap (while they last)
- High power output (high energy density)
- Variety of engines and devices use them directly and easily
- Extensive distribution network is in place

▶ Disadvantages of fossil fuels

- Will run out
- Pollute the environment
- Contribute to greenhouse effect by releasing greenhouse gases into atmosphere

In the overall considerations over choice of fuel, one must take into account the cost of transporting the fuel from its place of production to the place of distribution. Fossil fuels have generally high costs because the mass and volume of the fuel tend to be large. Similarly, one needs extensive storage facilities. Fossil fuels, especially oil, pose serious environmental problems due to leakages at various points along the production–distribution line.

Nuclear power

Nuclear **fission** is the process in which a heavy nucleus splits into lighter nuclei. The details of the process and the methods used to calculate the energy produced have been presented in detail in Chapter 6.3, which you should review.

Nuclear reactors

A nuclear reactor is a machine in which nuclear reactions take place, producing energy. The **fuel** of a nuclear reactor is typically uranium-235. The isotope of uranium that is most abundant in nature is uranium-238. Natural uranium contains only about 0.7% of uranium-235. The uranium fuel in a reactor is thus **enriched**, i.e. is made to contain more uranium-235, about 3% or even higher. When a nucleus of uranium-235 captures a neutron, it turns into uranium-236, which then decays, releasing energy and more neutrons. In addition to the common reaction discussed in Chapter 6.3, we also have the reaction:

$$^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{236}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2^{1}_{0}\mathrm{n}$$

These are examples of *induced* fission. The fission does not proceed by itself – neutrons must initiate it. (Some nuclei undergo *spontaneous* fission, i.e. no neutrons are necessary to initiate it, but this is rare.)

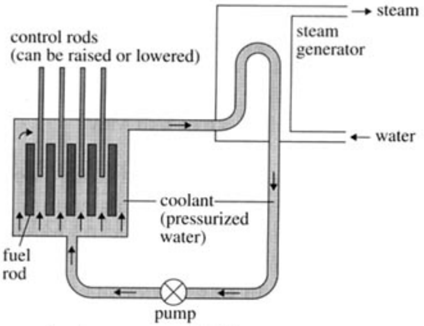
The neutrons produced can be used to collide with other nuclei of uranium-235 in the reactor, producing more fission, more energy and more neutrons. The reaction is thus self-sustaining; it is called a **chain reaction.** For the chain reaction to get going, a certain minimum mass of uranium-235 must be present, otherwise the neutrons escape without causing further reactions – this is called the **critical mass**. Uranium-235 will only capture neutrons if the neutrons are not too fast. The neutrons produced in the chain reaction are much too fast to be captured by uranium-235 (they have typical kinetic energies of about 1 MeV whereas to be absorbed the kinetic energy must be less than about 1 eV) and must therefore be slowed down.

The slowing down of neutrons is achieved through collisions of the neutrons with atoms of the **moderator**, a material surrounding the **fuel rods** (the tubes containing uranium-235). The moderator material can be graphite or water, for example. The rate of the reaction is determined by the number of neutrons available to be captured by uranium-235. Too few neutrons would result in the reaction stopping, while too many neutrons would lead to an uncontrollably large release of energy.

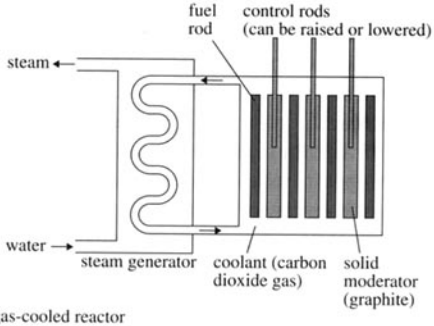
Thus **control rods**, i.e. a material that can absorb excess neutrons whenever this is necessary, are introduced into the moderator. The control rods can be removed when not needed and reinserted when necessary again. The control rods ensure that the energy from the nuclear reactions is released in a slow and controlled way as opposed to the uncontrolled release of energy that would take place in a nuclear weapon.

Schematic diagrams of the cores of two types of nuclear reactors are shown in Figure 1.6.

The energy released in the reaction is in the form of kinetic energy of the produced neutrons (and gamma ray photons). This kinetic energy is converted into thermal energy (in the



(a) pressurized water reactor (PWR)



(b) gas-cooled reactor

Figure 1.6 Schematic diagrams of two types of fission reactors.

moderator) as the neutrons are slowed down by collisions with the moderator atoms. A coolant (for example, water or liquid sodium) passing through the moderator can extract this energy, and use it in a heat exchanger to turn water into steam at high temperature and pressure. The steam can then be used to turn the turbines of a power station, finally producing electricity. These energy transformations are summarized in Figure 1.7.

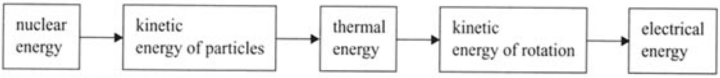


Figure 1.7 The main energy transformations that take place in a nuclear power station.

Plutonium production

The fast neutrons produced in a fission reaction may be used to bombard uranium-238 and produce plutonium-239. This isotope of plutonium does not occur naturally. The reactions are:

$$^{1}_{0}\mathrm{n} + ^{238}_{92}\mathrm{U} \rightarrow ^{239}_{92}\mathrm{U}$$

$$^{239}_{92}\mathrm{U} \rightarrow ^{239}_{93}\mathrm{Np} + ^{0}_{-1}\mathrm{e} + \overline{\nu}$$

$$^{239}_{93}\mathrm{Np} \rightarrow ^{239}_{94}\mathrm{Pu} + ^{0}_{-1}\mathrm{e} + \overline{\nu}$$

The importance of these reactions is that non-fissionable material (uranium-238) is being converted to fissionable material (plutonium-239) as the reactor operates. The plutonium-239 produced can then be used as the nuclear fuel in other reactors. (It can also be used in the production of nuclear weapons.)

Problems with nuclear reactors

The spent fuel in a nuclear reactor together with the products of the reactions are all highly radioactive, with long half-lives. The problem of how to dispose of this material safely is a serious disadvantage of the fission process in commercial energy production. At present, this material is buried deep underground in containers that are supposed to avoid leakage to the outside. In addition, there is always the possibility of an accident due to uncontrolled heating of the moderator. This might increase the temperature (in the case of a graphite moderator, it would also start a fire) and hence the pressure in the cooling pipes, resulting in an explosion. (This would be a conventional explosion – the reactor cannot explode in the way a nuclear weapon does.) In this case, radioactive material would leak from the sealed core of a reactor, dispersing radioactive material into the environment. Even worse, it may lead to the meltdown of the entire core. These are serious concerns with nuclear fission as a source of commercial power. On the positive side, nuclear power does not produce large amounts of greenhouse gases.

The nuclei produced in a fission reaction are typically unstable and decay usually by beta decay. The beta decay produces an additional amount of energy. Even if the reactor is shut down, production of thermal energy continues because of the beta decay of the product nuclei. The energy produced in this way is enough to melt the entire core of the reactor if the cooling system breaks down.

An additional worry about nuclear reactors is that the fissionable material produced (for example, plutonium-239) can be recovered and be used in a nuclear weapons programme.

Uranium mining

Like all forms of mining, uranium mining is dangerous, and in fact even more so. Uranium produces radon gas, a known strong carcinogen. Inhalation of this gas as well as of radioactive dust particles is a major hazard in the uranium mining business. Mine shafts require good ventilation and must be closed to avoid direct contact with the atmosphere. The disposal of waste material from the mining processes is also a problem, since the material is radioactive.

▶ Advantages of nuclear energy

- High power output
- Large reserves of nuclear fuels
- Nuclear power stations do not produce greenhouse gases

▶ Disadvantages of nuclear energy

- Radioactive waste products difficult to dispose of
- Major public health hazard should ‘something go wrong’
- Problems associated with uranium mining
- Possibility of producing materials for nuclear weapons

Nuclear fusion

A typical energy-producing nuclear **fusion** reaction is

$$^{2}_{1}\mathrm{H} + ^{3}_{1}\mathrm{H} \rightarrow ^{4}_{2}\mathrm{He} + ^{1}_{0}\mathrm{n} + 17.6\ \mathrm{MeV}$$

In this reaction, deuterium and tritium (two isotopes of hydrogen) fuse to form a helium nucleus and a neutron plus energy. Deuterium ($^{2}_{1}\mathrm{H}$) can be obtained by separating it from ordinary hydrogen in water using electrolysis. Tritium ($^{3}_{1}\mathrm{H}$) is obtained by bombarding lithium with neutrons. There are ample supplies of both water and lithium.

The problem with fusion is that, since they are both positively charged, the reacting nuclei repel. Thus, in order to get them close enough to each other for the reaction to take place, high temperatures must be reached. In this way the kinetic energy of the nuclei can be used to overcome the electrical repulsion. The temperatures required are of the order of 10^8 K. At this temperature, hydrogen atoms are ionized and so we have a **plasma** (a mixture of positive nuclei and electrons). The hot plasma must be confined in such a way that it does not come into contact with anything else. This is because contact with other materials would result in both (a) a reduction of temperature and (b) contamination of the plasma with other materials. These two effects would cause the fusion reaction to stop. The plasma is therefore confined magnetically in a *tokamak* (a Russian word for toroidal magnetic chamber) machine. This has specially designed magnetic fields that allow the plasma to move around magnetic field lines without touching the container walls.

Energy must be supplied to the fusion process to reach the high temperatures required. It has not yet been possible to produce more energy out of fusion than has first been put in, for sustained periods of time. For this reason, fusion as a source of commercially produced energy is not yet feasible. There are also technical problems with using the energy produced in fusion to produce electricity.

Compared to nuclear fission, nuclear fusion has the advantage of plentiful fuels, substantial amounts of energy produced, and much fewer problems with radioactive waste.

Solar power

The nuclear fusion reactions in the sun send out an incredible, and practically inexhaustible, amount of energy, at a rate of about 3.9×10^{26} W. This means that, on average, the earth receives about 1400 W per square metre of the surface of the outer atmosphere. Some of this radiation is reflected back into space, some is trapped by the atmosphere's gases, and about 1000 W m^{-2} (1 kW m^{-2}) is received on the surface of the earth. This amount assumes direct sunlight on a clear day and thus is the *maximum* that can be received at any one time. Averaged over a 24-hour time period, the intensity of sunlight is about 340 W m^{-2}. This high-quality, free and inexhaustible energy can be put to various uses.

Active solar devices

An early application of solar energy has been in what are called 'active solar devices'. In these, sunlight is used directly to heat water or air for heating in a house, for example. The collecting surface is usually flat and covered by glass for protection; the glass should be coated to reduce reflection. A blackened surface below the glass collects sunlight, and water circulating in pipes underneath gets heated. This hot water can then be used for household purposes, such as in bathrooms (the heated water is kept in well-insulated containers) or, with the help of a pump, it can circulate through a house, providing a heating effect.

In other schemes, the pipes can be exposed to sunlight directly, in which case they are blackened to increase absorption (Figure 1.8). The surface underneath the pipes is then reflecting so that more radiation enters the pipes. Such a collector works not only with direct sunlight but also with diffuse light, e.g. on partially cloudy days.

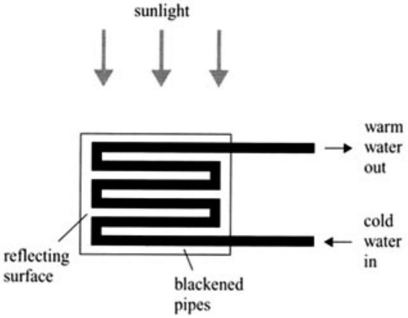


Figure 1.8 A device for collecting sunlight. The water in the pipes becomes heated and can be put to use.

These simple collectors are cheap and are usually put on the roof of a house. Their disadvantage is that they tend to be bulky and cover too much space.

More sophisticated collectors include a concentrator system in which the incoming solar radiation is focused, for example by a parabolic mirror, before it falls on the collecting surface. Such systems can heat water to much higher temperatures (500 °C to 2000 °C) than a simple flat collector. These high temperatures can be used to turn water into steam, which can drive a turbine, producing electricity (Figure 1.9). Obviously, back-up systems must be available in case of cloudy days, etc.

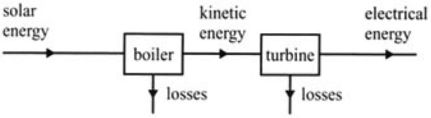


Figure 1.9 Energy flow diagram for electricity production by solar collectors.

Photovoltaic cells

A promising method for producing electricity from sunlight is that provided by photovoltaic cells. The photovoltaic cell was developed in 1954 at Bell Laboratories and is used extensively in the space programme. A photovoltaic cell converts sunlight into DC current at an efficiency of about 30% at present. In the past, the major cost of photovoltaic systems has been the cost of manufacturing the actual cell, but this cost is decreasing. From a cost of about \$100 per watt of power produced per cell, the price is now less than \$4. Adding on the price of related equipment, estimated at \$2 per watt, and used at a site with medium sunshine, 400 W m^{-2}, and taking into account a lifetime of the cell of about 20 years, this works out to a cost per kWh only slightly higher than that produced by diesel-powered generators.

The actual workings of the photovoltaic cell depend on the physics of semiconductors. However, it must be understood that the photovoltaic principle of electricity generation from sunlight is not the same as the photoelectric effect, where sunlight falling on certain surfaces also produces electric current. In the photoelectric effect, the electrons are actually ejected from the metal; whereas in the photovoltaic phenomenon, electrons, having absorbed photons of the right energy, make a transition from the valence band energy levels across the gap and into the conduction band energy levels (Figure 1.10).

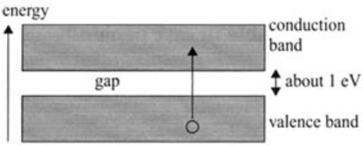


Figure 1.10 Energy level diagram for a silicon semiconductor.

As the price of photovoltaic systems drops, they are bound to become more dominant in electricity production around the world. Already, in places far from major power grids, their use is more economical than grid expansion, and these systems can usefully be used to power small remote villages, pump water in agriculture, power warning lights, etc. Their environmental ill-effects are practically zero, with the exception of chemical pollution at the place of their manufacture.

▶ **Advantages of solar energy**
- 'Free'
- Inexhaustible
- Clean

▶ **Disadvantages of solar energy**
- Works during the day only
- Affected by cloudy weather
- Low power output
- Requires large areas
- Initial costs high

The solar constant

The sun's total power output is $P = 3.9 \times 10^{26}$ W (this is also known as the sun's luminosity). On earth, we receive only a very small fraction of this total power output. The average distance between the sun and the earth is $r = 1.50 \times 10^{11}$ m. The sun's power is distributed uniformly over the surface of an imaginary sphere of radius $r = 1.50 \times 10^{11}$ m.

The power that is collected by area A (Figure 1.11) is the fraction $\frac{A}{4\pi r^2}$ of the total power P. (Note that $4\pi r^2$ is the surface area of the imaginary sphere.) The power through the area A is simply $P\frac{A}{4\pi r^2}$.

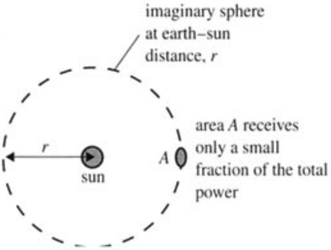


Figure 1.11.

▶ The *power per unit area* received at a distance r from the sun is called the intensity, I, and so

$$I = \frac{P}{4\pi r^2}$$

This amounts to about 1400 W m^{-2}, and is known as the *solar constant*. It is the power received by one square metre placed normally to the path of the incoming rays a distance of 1.50×10^{11} m from the sun.

This actual amount received varies somewhat due to the fact that the power output of the sun is not entirely constant. This gives variations of ±1.5%. In addition, the earth does not keep a constant distance from the sun (the orbit is slightly elliptical) and this gives additional variations of ±4.0%. To find the radiation received on the earth's *surface*, we must take into account reflection of the radiation from the atmosphere and the earth's surface itself, latitude, angle of incidence and average between day and night.

It is useful to define the total amount of energy received by one square metre of the earth's surface in the course of one day. This is called the **daily insolation**. Figure 1.12 shows the daily insolation at two different latitudes as a function of time. The curve with the big dip corresponds to a latitude of 60°. The other is for a latitude of 36°. At zero latitude (the equator), the insolation is almost constant at about 25 MJ m^{-2} day^{-1}.

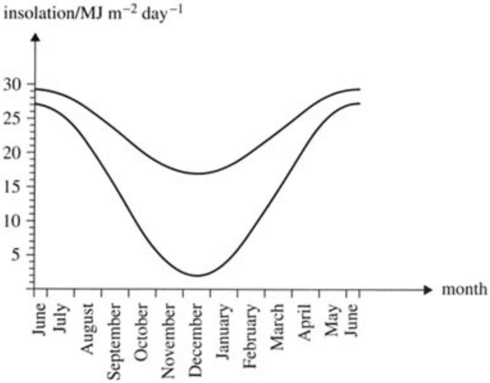


Figure 1.12 The daily insolation at two latitudes in the northern hemisphere. The curve with the big dip corresponds to a latitude of 60°. The other is for a latitude of 36°.

The reduction of the daily insolation in the winter for high latitudes can be explained by the shorter length of daylight and the oblique incidence of light (Figure 1.13).

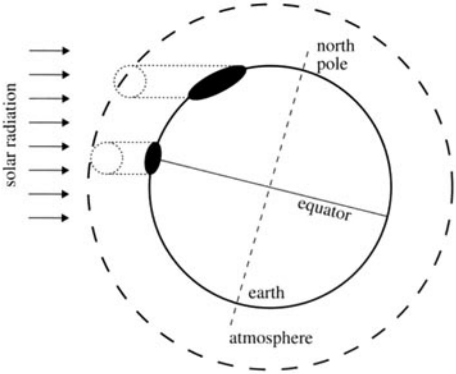


Figure 1.13 At higher latitudes, the energy is spread over a larger area and so the intensity is less. In addition, the radiation has to go through a greater depth of atmosphere and so some of the energy is absorbed.

Hydroelectric power

Hydropower, the power derived from moving water masses, is one of the oldest and most established of all renewable energy sources. This highly site-dependent energy source is capable of producing very cheap electricity. Its use has expanded very rapidly, with power output from hydroelectric plants doubling every 15 years. Turbines driven by falling or moving water have a long working life without major maintenance costs. Also, despite the high costs of the initial construction, hydropower is very promising for many parts of Africa and South America. It is widely used in Norway. Hydropower stations are, however, associated with massive changes in the ecology of the area surrounding the plants. To create a reservoir behind a newly constructed dam, a vast area of land must be flooded.

The principle behind hydropower is simple. Consider a mass *m* of water that falls down a vertical height h (Figure 1.14). The potential energy of the mass is mgh, and this gets converted into kinetic energy when the mass descends the vertical distance h. The mass is given by $\rho\Delta V$, where ρ is the density of water ($1000\ \text{kg m}^{-3}$) and ΔV is the volume it occupies (see Figure 1.15).

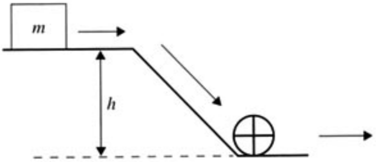


Figure 1.14 Water falling from a vertical height h has its potential energy converted into kinetic energy, which can be used to drive turbines.

The rate of change of this potential energy, i.e. the power P, is given by the change in potential energy divided by the time taken for that change, so

$$P = \frac{mgh}{\Delta t} = \frac{\rho \Delta V gh}{\Delta t} = \rho \frac{\Delta V}{\Delta t} gh$$

The quantity $Q = \frac{\Delta V}{\Delta t}$ is known as the volume flow rate (volume per second) and so

$$P = \rho Q g h$$

Within a time equal to Δt, the mass of water that will flow through the tube (Figure 1.15) is $m = \rho\Delta V = \rho Q \Delta t$.

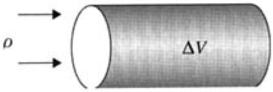


Figure 1.15.

This is the power available for generating electricity (or to convert into some other mechanical form) and it is thus clear that hydropower requires large volume flow rates, Q, and large heights, h.

Example question

Q2

Find the power developed when water in a stream with a flow rate $50\ \text{L s}^{-1}$ falls from a height of 15 m.

Answer

Applying the power formula, we find (remembering to do all the conversions)

$$P = \rho Q g h = 1000 \times (50 \times 10^{-3}) \times 9.8 \times 15$$
$$= 7.4\ \text{kW}$$

A number of different schemes are available for extracting the power of water. Water can be stored in a *lake*, which should be at as high an elevation as possible to allow for energy release when the water is allowed to flow to lower heights. In a *pump storage system*, the water that flows to lower heights is again pumped back to its original height by using the generators of the plant as motors to pump the water. Obviously, to do this requires energy (more energy, in fact, than can be regained when the water is again allowed to flow to lower heights). This energy has to be supplied from other sources of electrical energy. But it is the only way to *store* energy on a large scale for use when demand is high. In other words, *excess* electricity from somewhere else can be provided to the plant to raise the water so that energy can be produced *later* when it is needed. Finally, there are schemes that take advantage of the tides, *tidal storage systems*. The general idea here is to have the flow of water during a tide turn turbines, producing electricity.

▶ Advantages of hydroelectric energy
- 'Free'
- Inexhaustible
- Clean

▶ Disadvantages of hydroelectric energy
- Very dependent on location
- Requires drastic changes to environment
- Initial costs high

Wind power

This ancient method for extracting energy is particularly useful for isolated small houses and agricultural use, where small wind turbines extracting 3 kW of power from the wind can provide all the energy needed for simple living conditions. Small wind turbines have vanes no larger than about 1 m long. Modern large wind turbines are capable of producing up to a few megawatts of power and, of course, they tend to be big, with vanes larger than 30 m.

Wind power devices have no adverse effects (though there is some evidence that low-frequency sound emitted during the operation of wind turbines affects people's sleeping habits). However, a very large number of them in wind parks is not an attractive sight to many people, and there is a noise problem. The cost of wind energy conversion systems varies from about \$500 to \$5000 per kilowatt of power produced. The blades are susceptible to stresses in high winds, and damage due to metal fatigue frequently occurs. The design must also take into account gale-force winds, which may be very rare for a particular site, but would certainly result in very serious damage to an inadequately designed system. Generally, about one-quarter of the power carried by the wind can be converted into electricity (Figure 1.16).

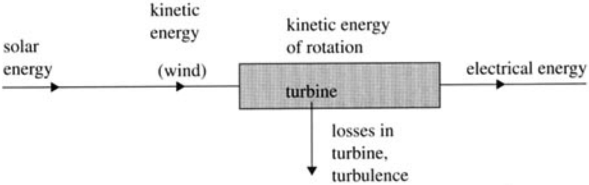


Figure 1.16 Energy flow diagram for wind energy extraction.

Wind speed is the crucial factor for these systems, the power extracted being proportional to the cube of the wind speed. Wind speeds of up to about 4 m s^{-1} are not particularly useful for energy extraction. Serious power production from wind occurs at speeds from 6 to 14 m s^{-1}. The dependence of the power on the area of the blades and the cube of the wind speed can be understood as follows (Figure 1.17).

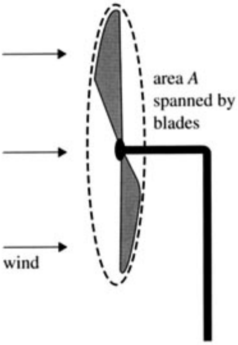


Figure 1.17 A horizontal axis wind turbine with two blades.

Let us consider the mass of air that can pass through a tube of cross-sectional area A with velocity v in time Δt (Figure 1.18). Let ρ be the density of air. Then the mass enclosed in a tube of length $v\,\Delta t$ is $\rho A v\,\Delta t$. This is the mass that will exit the right end of the tube *within* a time interval equal to Δt. The kinetic energy of this mass of air is thus

$$\frac{1}{2}(\rho A v\,\Delta t)v^2 = \frac{1}{2}\rho A \Delta t v^3$$

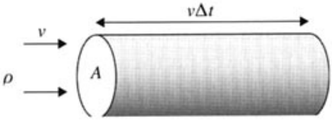


Figure 1.18 The mass of air within this cylinder will exit the right end within a time of Δt.

▶ The kinetic energy per unit time is the power, and so dividing by Δt we find

$$P = \frac{1}{2}\rho A v^3$$

which shows that the power carried by the wind is proportional to the cube of the wind speed and proportional to the area spanned by the blades.

If this stream of air meets a wind turbine, then A stands for the area presented to the air stream by the blades of the wind turbine. The power that can be extracted is thus

$$P = C_p \frac{1}{2} \rho A v^3$$

where C_p is known as the power coefficient. It is simply an efficiency factor that determines how much of the available wind power the wind turbine can extract. Theoretically, C_p is between 0.35 and 0.45.

Assuming a wind speed of 12 m s^{-1}, an air density of 1.2 kg m^{-3} and an efficiency coefficient of 0.40, we find

$$\begin{aligned}\frac{P}{A} &= C_p \frac{1}{2} \rho v^3 \\ &= 0.40 \times \frac{1}{2} \times 1.2 \times 12^3 \\ &= 4.1 \times 10^2 \text{ W m}^{-2}\end{aligned}$$

as the theoretical power extracted per unit wind turbine area. A 2 m^2 wind turbine area will thus extract about 820 W of power from the wind.

Doubling the wind turbine area doubles the power extracted, but doubling the wind speed increases the power (in theory) by a factor of eight. In practice, frictional and other losses (mainly turbulence) result in a smaller power increase. The calculations above also assume that all the wind is actually *stopped* by the wind turbine, extracting all of the wind's kinetic energy, which in practice is not the case.

▶ **Advantages of wind power**

- The source is the wind and so 'free'
- For practical purposes it is inexhaustible
- Clean, without carbon emissions
- Ideal for remote island locations

▶ **Disadvantages of wind power**

- Works only if there is wind – not dependable
- Low power output
- Aesthetically unpleasant (and noisy)
- Best locations far from large cities
- Maintenance costs high

Wave power

It has been realized that deep-water, long-wavelength sea waves carry a lot of energy. Water waves are very complex and belong to a class of waves called *dispersive*, i.e. the speed of the wave depends on the wavelength.

▶ A water (sea) wave of amplitude A carries an amount of power *per unit length of its wavefront* equal to

$$\frac{P}{L} = \frac{\rho g A^2 v}{2}$$

where ρ is the density of water and v stands for the *speed of energy transfer* in the wave.

For waves of amplitude 1.5 m and wave energy transfer speed $v = 4.0$ m s^{-1}, the formula above gives (here it is worth paying attention to the units)

$$\begin{aligned}\frac{P}{L} &= \frac{1}{2} \rho g A^2 v \\ &= \frac{1}{2} \times 10^3 \times 9.8 \times 1.5^2 \times 4.0 \\ &= 44 \text{ kW m}^{-1}\end{aligned}$$

The units are:

$$\begin{aligned}&(\text{kg m}^{-3})(\text{N kg}^{-1})(\text{m})^2(\text{m s}^{-1}) \\ &= (\text{N m})\,\text{s}^{-1}\,\text{m}^{-1} \\ &= (\text{J})\,\text{s}^{-1}\,\text{m}^{-1} \\ &= \text{W m}^{-1}\end{aligned}$$

This is a substantial amount of power.

Supplementary material

In advanced books you may see this formula also written with a denominator of 4 rather than 2, i.e.

$$\frac{P}{L} = \frac{\rho g A^2 c}{4}$$

Here $c = \frac{\lambda}{T}$ is the speed of the wave, which, because the wave is dispersive, is not equal to the energy transfer speed v; in fact $c = 2v$. So the wave speed in the example above is $c = 2 \times 6.25\,\mathrm{m\,s^{-1}} = 12.5\,\mathrm{m\,s^{-1}}$. This is a technical point that we will not discuss further.

Many devices have been proposed to extract the power out of waves. The one to be discussed here is called the oscillating water column (Figure 1.19). As a crest of the wave approaches the cavity in the device, the column of water in the cavity rises and so pushes the air above it upwards. The air passes through a turbine, turning it, and is then released into the atmosphere. As a trough of the wave approaches the cavity, the water in the cavity falls and thus draws in air from the atmosphere, which again turns the turbine.

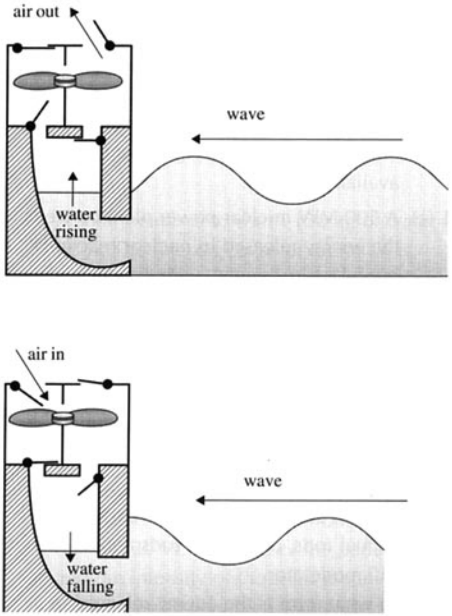


Figure 1.19 An oscillating water column (OWC) plant.

Wave patterns are irregular in wave speed, amplitude and direction, and it is difficult to achieve reasonable efficiency of wave devices over all the variables. For many wave devices, it is difficult to couple the low frequency of the water waves (typically 0.1 Hz) to the much higher generator frequencies (50–60 Hz) required for electricity production. The OWC solves this problem. The great advantage of the OWC device is that the speed of the air through the column can be increased by adjusting the diameter of the valves though which the air passes. In this way very high air speeds can be attained, thus coupling the low-frequency water waves with the high-frequency turbine motion.

▶ Advantages of wave power

- The source is waves and so 'free'
- Reasonable energy density
- For practical purposes it is inexhaustible
- Clean, without carbon emissions

▶ Disadvantages of wave power

- Works only in areas with large waves
- Irregular wave patterns make it difficult to achieve reasonable efficiency
- Difficult to couple low-frequency water waves with high-frequency turbine motion
- Maintenance and installation costs very high
- Transporting the produced power to consumers involves high costs
- Devices must be able to withstand hurricane and gale-force storms

Questions

1 Consider a scheme in which thermal energy is extracted from the ocean. Some of the extracted energy is used to perform mechanical work (run the ship) and the rest is discarded back into the ocean. Why will this not work?

2 Explain what is meant by *degradation of energy*. Give one example of energy degradation.

3 (a) Define *energy density* of a fuel.
   (b) Estimate the energy density of water that falls from a waterfall of height 75 m and is used to drive a turbine.

4 A power plant produces 500 MW of power.
   (a) How much energy is produced in one second? Express your answer in (i) joules, (ii) kWh and (iii) MWh.
   (b) How much energy (in joules) is produced in one year?

5 A power plant operates in four stages. The efficiency in each stage is 80%, 40%, 12% and 65%.
   (a) What is the overall efficiency of the plant?
   (b) Make a Sankey diagram for the energy flow in this plant.

6 A coal power plant with 30% efficiency burns 10 million kilograms of coal a day. (Take the heat of combustion of coal to be 30 MJ kg^{-1}.)
   (a) What is the power output of the plant?
   (b) At what rate is thermal energy being discarded by this plant?
   (c) If the discarded thermal energy is carried away by water whose temperature is not allowed to increase by more than 5 °C, calculate the rate at which water must flow away from the plant.

7 One litre of gasoline releases 35 MJ of energy when burned. The efficiency of a car operating on this gasoline is 40%. The speed of the car is 9.0 m s^{-1} when the power developed by the engine is 20 kW. Calculate how many kilometres the car can go with one litre of gasoline when driven at this speed.

8 A coal-burning power plant produces 1.0 GW of electricity. The overall efficiency of the power plant is 40%. Taking the energy density of coal to be 30 MJ kg^{-1}, calculate the amount of coal that must be burned in one day.

9 In the context of nuclear fission reactors, state what is meant by
   (a) uranium enrichment;
   (b) moderator;
   (c) critical mass.

10 (a) Calculate the energy released in the fission reaction

$$^{1}_{0}\mathrm{n} + {}^{235}_{92}\mathrm{U} \rightarrow {}^{236}_{92}\mathrm{U} \rightarrow {}^{140}_{54}\mathrm{Xe} + {}^{94}_{38}\mathrm{Sr} + 2\,{}^{1}_{0}\mathrm{n}$$

   (b) How many fission reactions per second must take place if the power output is 200 MW? (The atomic masses are: uranium-235, ${}^{235}_{92}\mathrm{U} = 235.043\,923$ u; xenon-140, ${}^{140}_{54}\mathrm{Xe} = 139.921\,636$ u; strontium-94, ${}^{94}_{38}\mathrm{Sr} = 93.915\,360$ u; neutron, ${}^{1}_{0}\mathrm{n} = 1.008\,665$ u.)

11 The energy released in a typical fission reaction involving uranium-235 is 200 MeV.
   (a) Calculate the energy density of uranium-235.
   (b) How much coal (heat of combustion 30 MJ kg^{-1}) must be burned in order to give the same energy as that released in nuclear fission with 1 kg of uranium-235 available?

12 (a) A 500 MW nuclear power plant converts the energy released in nuclear reactions into electrical energy with an efficiency of 40%. Calculate how many fissions of uranium-235 are required per second. Take the energy released per reaction to be 200 MeV.
   (b) What mass of uranium-235 is required to fission per second?

13 (a) Make a schematic diagram of a fission reactor, explaining the role of (i) fuel rods, (ii) control rods and (iii) moderator.
   (b) In what form is the energy released in a fission reactor?

14 By looking up appropriate sources, write an essay about the problem of radioactive waste disposal.

15 Distinguish between a solar panel and a photovoltaic cell.

16 The typical energy of photons in the visible spectrum is 2 eV. Explain why a semiconductor with an energy gap between the valence and conduction bands of more than 2 eV would not be suitable in a photovoltaic cell.

17 Sunlight of intensity 700 W m^{-2} is captured with 70% efficiency by a solar panel, which then sends the captured energy into a house with 50% efficiency.

(a) If the house loses thermal energy through bad insulation at a rate of 3.0 kW, find the area of the solar panel needed in order to keep the temperature of the house constant.

(b) Make a Sankey diagram for the energy flow.

18 A solar heater is to heat 300 L of water initially at 15 °C to a temperature of 50 °C in a time of 12 hours. The amount of solar radiation falling on the collecting surface of the solar panel is 240 W m^{-2} and is collected at an efficiency of 65%. Calculate the area of the collecting panel that is required.

19 A solar heater is to warm 150 kg of water by 30 K. The intensity of solar radiation is 6000 W m^{-2} and the area of the panels is 4.0 m^2. The specific heat capacity of the water is 4.2×10^3 J kg^{-1} K^{-1}. Estimate the time this will take, assuming a solar panel efficiency of 60%.

20 The graph in Figure 1.20 shows the variation with incident solar power P of the temperature of a solar panel used to heat water when thermal energy is extracted from the water at a rate of 320 W. The area of the panel is 2.0 m^2 and the intensity of the solar radiation incident on the panel is 400 W m^{-2}. Calculate

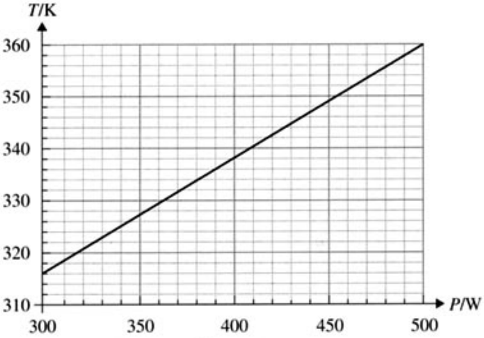


Figure 1.20 For question 20.

(a) the temperature of the water;

(b) the power incident on the panel;

(c) the efficiency of the panel.

21 The graph in Figure 1.21 shows the power curve of a wind turbine as a function of the wind speed. If the wind speed is 10 m s^{-1}, calculate the energy produced in the course of one year, assuming that the wind blows at this speed for 1000 hours in the year.

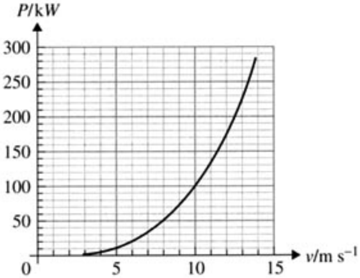


Figure 1.21 For question 21.

22 State the expected increase in the power extracted from a wind turbine when

(a) the length of the blades is doubled;

(b) the wind speed is doubled;

(c) both the length of the blades and the wind speed are doubled.

(d) Outline reasons why the actual increase in the extracted power will be less than your answers.

23 Wind of speed v is incident on the blades of a wind turbine. The blades present the wind with an area A.

(a) Deduce that the maximum theoretical power that can be extracted is given by

$$P = \frac{1}{2}\rho A v^3$$

(b) State any assumptions made in deriving the relation in (a).

24 Air of density 1.2 kg m^{-3} and speed 8.0 m s^{-1} is incident on the blades of a wind turbine. The radius of the blades is 1.5 m. Immediately after passing through the blades, the wind speed is reduced to 3.0 m s^{-1} and the density of air is 1.8 kg m^{-3}. Calculate the power extracted from the wind.

25 Calculate the blade radius of a wind turbine that must extract 25 kW of power out of wind of speed 9.0 m s^{-1}. The density of air is 1.2 kg m^{-3}. State any assumptions made in this calculation.

26 Find the power developed when water in a waterfall with a flow rate of 500 L s^{-1} falls from a height of 40 m.

27 Water falls from a vertical height h at a flow rate (volume per second) Q. Deduce that the maximum theoretical power that can be extracted is given by $P = \rho Q g h$.

28 A student explaining pumped storage systems says that the water that is stored at a high elevation is allowed to move lower, thus producing electricity. Some of this electricity is used to raise the water back to its original height, and the process is then repeated. What is wrong with this statement?

29 (a) Supply the details for the derivation of the equation

$$\frac{P}{L} = \frac{\rho g A^2 v}{2}$$

for a wave with a square profile.

(b) Calculate the power per unit wavefront length that can be obtained from deep-sea waves of amplitude 5.0 m and wave speed $v = 4.8\,\mathrm{m\,s^{-1}}$.

(c) What wavefront length is required for a total power output of 1.0 MW.

30 Describe the operation of an oscillating water column (OWC) device. State the main advantage of the OWC device.

31 Make an annotated energy flow diagram showing the energy changes that are taking place in each of the following:

(a) a conventional electricity-producing power station using coal;

(b) a hydroelectric power plant;

(c) an electricity-producing wind turbine;

(d) an electricity-producing nuclear power station.

HL only

32 Sunlight of intensity 800 W m^{-2} is captured by a tank containing 100 kg of water with an efficiency of 80%. The tank is rectangular in shape and has dimensions $1.0 \times 1.0 \times 0.10\,\mathrm{m}^3$. It has walls of thickness 5.0 mm. The surrounding air has a temperature of 20 °C. Assume that the tank is well insulated from all sides except the top surface (of area 1.0 m^2). The material of the tank has a thermal conductivity of $k = 0.30\,\mathrm{W\,m^{-1}\,K^{-1}}$, its density is 1200 kg m^{-3} and its specific heat capacity is 450 J kg^{-1} K^{-1}.

The rate of flow of thermal energy through a surface of area A and thickness x separated by temperatures T_1 and T_2 is given by

$$\frac{\Delta Q}{\Delta t} = kA\frac{T_1 - T_2}{x}$$

(a) Calculate the mass of the tank.

(b) By equating the energy received from the sunlight to the thermal energy lost by conduction to the surrounding air, estimate the final temperature of the water.

(c) Find the heat capacity, C, of the tank–water system.

(d) Show that the temperature T of the water in °C is increasing at a rate $\frac{\Delta T}{\Delta t}$ that can be found from the equation

$$C\frac{\Delta T}{\Delta t} = AI_{\text{in}} - kA\frac{T-20}{x}$$

where I_{in} is the intensity of sunlight captured by the tank, C is the heat capacity of the system, k is the thermal conductivity of the tank, A is the area of the top surface of the tank, and x is the thickness of the tank wall.

(e) Evaluate the rate of temperature increase when the temperature is the average of the initial temperature of 20 °C and the final temperature you found in part (b).

(f) Assuming that the temperature is increasing at this rate, calculate how long it will take the water to reach its final temperature.

Chapter 6.6

1 $6.6 \times 10^{7}\ \text{m s}^{-1}$.

2 See Figure A63.

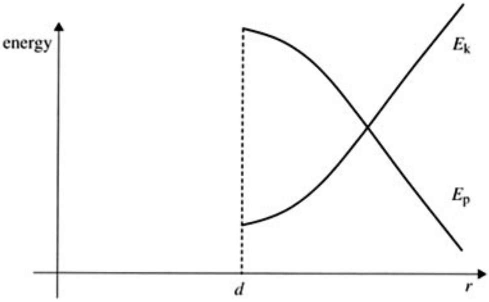


Figure A63.

3 (a) $3.0 \times 10^{4}\ \text{N C}^{-1}$; (b) 0.083 m; (c) 0.091 m.

6 (a) $0.231\ \text{s}^{-1}$; (b) (i) 4.78×10^{21};
(ii) 3.79×10^{21}; (iii) 3.01×10^{21}.

7 (a) 0.5; (b) 0.875; (c) 0.5.

8 3.66×10^{10} Bq.

9 1.10×10^{6} Bq.

10 4.20×10^{11}.

11 3.8×10^{9} yr.

12 4.11×10^{9} yr.

13 (b) About 7.2 min.

14 (a) 0.75; (b) 0.95; (c) 1.50.

18 (a) 1.5×10^{-15} m; (b) 4000 N; (c) 230.4 N;
(d) 1.86×10^{-34} N; (e) 1.24×10^{36}.

20 (a) ${}^{226}_{88}\text{Ra} \rightarrow {}^{226}_{88}\text{Ra} + {}^{0}_{0}\gamma$; (b) 1.83×10^{-11} m.

7 Energy, power and climate change

Chapter 7.1

3 (b) $7.4 \times 10^{2}\ \text{J kg}^{-1}$.

4 (a) (i) 5×10^{8} J; (ii) 140 kWh; (iii) 0.140 MWh.
(b) 1.6×10^{16} J.

5 (a) 2.5%.

6 (a) 1.0×10^{9} W; (b) 2.4×10^{9} W;
(c) $1.2 \times 10^{5}\ \text{kg s}^{-1}$.

7 6.3 km.

8 $7.2 \times 10^{6}\ \text{kg day}^{-1}$.

10 (a) 185 MeV or 2.96×10^{-11} J; (b) $6.77 \times 10^{18}\ \text{s}^{-1}$.

11 (a) $8.20 \times 10^{13}\ \text{J kg}^{-1}$; (b) 2.7×10^{6} kg.

12 (a) $3.9 \times 10^{19}\ \text{s}^{-1}$; (b) $1.5 \times 10^{-5}\ \text{kg s}^{-1}$.

17 (a) $12\ \text{m}^2$.

18 $6.5\ \text{m}^2$.

19 3.6 h.

20 (a) 339 K; (b) 800 W; (c) 0.40.

21 3.6×10^{11} J.

22 (a) Increases by a factor of 4.
(b) Increases by a factor of 8.
(c) Increases by a factor of 32.

24 2.0 kW.

25 4.3 m.

26 2.0×10^{5} W.

29 (b) 5.9×10^{5} W; (c) 1.7 m.

32 (a) 14 kg; (b) 31°C; (c) $4.3 \times 10^{5}\ \text{J K}^{-1}$;
(e) $7.5 \times 10^{-4}\ \text{K s}^{-1}$; (f) 3.9 hrs.

Chapter 7.2

1 (c) 1.8.

2 (b) 0.6.

3 278 K.

4 (a) $T \propto \frac{1}{\sqrt{d}}$; (b) 1.4 K.

5 $2.4\ \text{W m}^{-2}$.

7 (a) $(4.5 \pm 0.1) \times 10^{2}$ K.
(b) Similar curve that is overall higher with peak shifted to the left.

9 (b) 0.29; (c) $250\ \text{W m}^{-2}$; (d) 258 K.

10 (a) 172 h; (b) $4.5 \times 10^{24}\ \text{J K}^{-1}$; (c) 4×10^{7} s
(a bit more than a year).

11 (a) $T_{\text{Venus}} \approx 263$ K; $T_{\text{Mars}} \approx 217$ K.

12 (a) (i) $\left(\frac{1-\alpha}{t}\right)\frac{S}{4}$; (ii) $(1-t)\left(\frac{1-\alpha}{t}\right)\frac{S}{4}$;
(iii) $(1-\alpha)\frac{S}{4}$.
(b) $\left(\frac{1-\alpha}{t}\right)\frac{S}{4} = \sigma T^{4}$, giving $t = 0.63$.

13 (b) 10.

20 (a) 0.27.

23 Approximately 2 K increase in temperature.

29 (a) 3.3×10^{10} J; (b) no.

30 0.9 m.

31 $2 \times 10^{6}\ \text{km}^3$.

8 Digital technology

Chapter 8.1

1 (a) 11; (b) 1010; (c) 10010; (d) 11111.

2 (a) 6; (b) 12; (c) 5; (d) 30.